Example **Ed believes that a five-sided spinner is biased towards landing on 5. He spins the spinner 20 times and it lands on 5 ten times. Using a 5% level of significance, test the hypothesis that the spinner is biased towards landing on 5.**

1. Identify the population parameter. p = the probability of the spinner landing on 5

2. Formulate the null and alternative hypotheses for p.
The null hypothesis is that the spinner is not biased and the alternative hypothesis is that the spinner is more likely to land on 5.
$H_0: p = 0.2$
$H_1: p > 0.2$

3. State the test statistic X and its probability distribution under H_0.
Let X = number of times the spinner lands on a 5 in the sample. Under H_0, $X \sim B(20, 0.2)$.

4. State the significance level of the test. $\alpha = 0.05$

5. The observed value is 10, so find the probability of X being 10 or more, under the null hypothesis.

$$P(X \geq 10) = 1 - P(X < 10) = 1 - P(X \leq 9)$$
$$= 1 - 0.9974... = 0.0026 \text{ (4 d.p.)}$$
Since $0.0026 < 0.05$, the result is significant.

6. Write your conclusion.
There is evidence at the 5% level of significance to reject H_0 and to support Ed's claim that the spinner is biased towards landing on 5.

Exercise 2.2

Q1 Charlotte claims she can read Milly's mind. To test this claim, Milly thinks about a number from 1 to 5 while Charlotte attempts to read her mind. Charlotte is right on 4 out of 10 occasions.
a) Write down the population parameter and suitable null and alternative hypotheses.
b) Define the test statistic and write down its sampling distribution under the null hypothesis.
c) Are these results significant at a 5% level of significance?

Q2 Last year 45% of students said that the chicken dinosaurs in the school canteen were good value. After this year's price increase Ellen says fewer people think they are good value. She asked 50 people and found only 16 said that chicken dinosaurs were good value. Test Ellen's claim at the 10% level.

Q3 Pete's Driving School advertises that 70% of its clients pass the driving test at their first attempt. Hati and three of her friends failed. Four other friends did pass first time. She complained that the advertisement was misleading and that the percentage was actually lower. Test whether there is evidence to support Hati's complaint at the 1% level.

4.

In a certain bank, the probability that a phone call is in a queue for more than five minutes is 0.3.

A new telephone console is installed in order to improve efficiency, however when a sample of 15 calls is checked, 9 calls were found to be queuing for more than five minutes.

Test, at the 1% level of significance, whether there is evidence that more calls are now queuing for more than five minutes.

5.

The proportion of vegetarian orders in a restaurant is thought to be 15% .

During lunch on a given day, 7 diners out of 20 , ordered vegetarian food.

Test, at 5% level of significance, whether the proportion of diners who order vegetarian food is higher than 15% .

6.

A mayoral candidate, Hans Van Dyke, claims that 40% of the electoral will vote for him in the next election. In a recent opinion poll of 20 recently selected voters it was found that **only 4** people will vote for Hans Van Dyke.

a) Test, at the 5% level of significance whether, or not, the opinion poll supports Hans Van Dyke's claim.

In a second opinion poll of n randomly selected people, it was found that no one will be voting for Hans Van Dyke. As a result of this poll, Hans Van Dyke's claim is rejected at 1% significance.

b) Determine the smallest value of n .

Exercise 2.2

Q1 **a)** Let p be the probability of Charlotte guessing correctly. Then $H_0: p = 0.2$ and $H_1: p > 0.2$.

b) Let X be the number of times Charlotte is correct in the sample. Then under H_0, $X \sim B(10, 0.2)$.

c) $P(X \geq 4) = 0.1209 > 0.05$. So there is not significant evidence at the 5% level to reject H_0 in favour of Charlotte's claim.

Q2 There is significant evidence at the 10% level to reject H_0 in favour of Ellen's claim that fewer people think chicken dinosaurs are good value.

Q3 There is not significant evidence at the 1% level to reject H_0 in favour of H_1, so Hati's claim is not upheld at the 1% level.

Q4

X = NUMBER OF CALLS QUEUING FOR MORE THAN 5 MINUTES

$X \sim B(15, 0.3)$

$H_0 : p = 0.3$

$H_1 : p > 0.3$, WHERE p REPRESENTS THE PROPORTION OF ALL CALLS QUEUING MORE THAN FIVE MINUTES

TESTING AT 1% SIGNIFICANCE ON THE BASIS THAT $x = 9$

$$P(X \geq 9) = 1 - P(X \leq 8)$$
$$= 1 - 0.98475\ldots$$
$$= 0.0152$$
$$= 1.52\%$$
$$> 1\%$$

THERE IS NO SIGNIFICANT EVIDENCE THAT MORE CALLS ARE NOW QUEUING LONGER THAN FIVE MINUTES — THERE IS NO SUFFICIENT EVIDENCE TO REJECT H_0

Q5

X = NUMBER OF VEGETARIAN ORDERS

$X \sim B(20, 0.15)$

$H_0 : p = 0.15$

$H_1 : p > 0.15$

p IS THE PROPORTION OF VEGETARIAN ORDERS IN GENERAL

TESTING AT 5% SIGNIFICANCE ON THE BASIS $x = 7$

$$P(X \geq 7) = 1 - P(X \leq 6)$$
$$= 1 - 0.97806\ldots$$
$$= 0.0219\ldots$$
$$= 2.19\%$$
$$< 5\%$$

THERE IS SIGNIFICANT EVIDENCE THAT THE PROPORTION OF VEGETARIAN ORDERS IS HIGHER THAN 15%

THERE IS SIGNIFICANT EVIDENCE TO REJECT H_0

Q6

a) X = NUMBER OF HVD VOTERS

$X \sim B(20, 0.4)$

H_0: $p = 0.4$

H_1: $p < 0.4$, WHERE p IS THE PROPORTION OF HVD VOTERS IN GENERAL

↑ BECAUSE OF "ONLY 4", OTHERWISE $p \neq 0.4$

TESTING AT 5% SIGNIFICANCE, ON THE BASIS THAT $x = 4$

$P(X \leq 4) = 0.05095\ldots$

$= 5.10\%$

$> 5\%$

THERE IS NO SIGNIFICANCE TO SUGGEST THAT $p < 0.4$, SO H.V.D CLAIM IS JUSTIFIED

THERE INSUFFICIENT EVIDENCE TO REJECT H_0

b) WE NOW REQUIRE FOR $X \sim B(n, 0.4)$ THAT $P(X \leq 0) < 0.01$

BY TRIAL & IMPROVEMENT OR LOGARITHMS

$\rightarrow \binom{n}{0}(0.4)^0(0.6)^n < 0.01$

$\Rightarrow 1 \times 1 \times 0.6^n < 0.01$

$\Rightarrow 0.6^n < 0.01$

$\Rightarrow \log(0.6^n) < \log(0.01)$

$\Rightarrow n\log(0.6) < \log(0.01)$ ← DIVIDING BY A NEGATIVE QUANTITY

$\Rightarrow n > \dfrac{\log(0.01)}{\log(0.6)}$

$\Rightarrow n > 9.015\ldots$

$\therefore n = 10$